

Analysis 1, Summer 2024

**List 0**

*Algebra review*

1. Which of the following are true for all real values of the variables (or, at least, all real values for which both sides of the equation are defined)?

- |   |  |
|---|--|
| (a) $2x = x + x$                        | (h) $k^{-2} = \frac{1}{k^2}$                     |
| (b) $2(x + y) = 2x + y$                 | (i) $x^{a+2} = x^a \times x^2$                   |
| (c) $(x - y)^2 = x^2 - 2xy + y^2$       | (j) $\sqrt{a + b} = \sqrt{a} + \sqrt{b}$         |
| (d) $\frac{6 + a}{2} = 3 + \frac{a}{2}$ | (k) $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ |
| (e) $-(y + 2) = -y + 2$                 | (l) $\log(a + b) = \log(a) + \log(b)$            |
| (f) $-(a + b)^2 = (-a + b)^2$           | (m) $\log(a \cdot b) = \log(a) + \log(b)$        |
| (g) $x^3 + 3x = x + x$                  | (n) $\log(a \cdot b) = \log(a) \cdot \log(b)$    |

2. Compute the following values:

- |                      |                       |                     |
|----------------------|-----------------------|---------------------|
| (a) $\cos(30^\circ)$ | (g) $\sqrt{10000}$    | (m) $\log_9(3)$     |
| (b) $\cos(45^\circ)$ | (h) $10000^{1/2}$     | (n) $\log_k(1)$     |
| (c) $\cos(60^\circ)$ | (i) $\sqrt[4]{10000}$ | (o) $\ln(1)$        |
| (d) $\cos(\pi/3)$    | (j) $625^{-1/2}$      | (p) $\ln(\sqrt{e})$ |
| (e) $\cos(\pi/2)$    | (k) $\log_7(49)$      | (q) $\log_4(16^3)$  |
| (f) $\sin(8\pi/3)$   | (l) $\log_4(1024)$    |                     |

3. Re-write  $\frac{x^4}{\sqrt{x^{13}}}$  in the form  $x^\square$ . (That box is not a mistake in the file.)

4. Re-write each of the following in the form  $\square x^\square$ , or state that this is not possible. You can assume  $x > 0$  if necessary.

- |                       |                             |  |
|-----------------------|-----------------------------|--|
| (a) $x^3 \cdot x^7$   | (g) $(x^3)^7$               | (n) $\ln(e^x)$                           |
| (b) $x^3 x^7$         | (h) $3x^5 + (x^5)^2$        | (o) $\ln(e^{6x})$                        |
| (c) $x^3 + x^7$       | (i) $3x^{10} + (x^5)^2$     | (p) $e^{9\ln(x)}$                        |
| (d) $x^3 - x^7$       | (j) $\sqrt{x^4}$            | (q) $e^{9\ln(x)+1}$                      |
| (e) $x^3/x^7$         | (k) $\sqrt{x^6 + x^4}$      | (r) $e^{9\ln(x)} + 1$                    |
| (f) $\frac{x^3}{x^7}$ | (l) $\sqrt{x^6} + 8x^3$     | (s) $\frac{\sqrt{4e^{\ln(9x)+10}}}{e^5}$ |
|                       | (m) $x^3 \cdot \sqrt[3]{x}$ |  |

5. Re-write  $\log_{10}(2^9)$  using the natural logarithm ( $\ln$ ).

6. Expand each of the following:

- |                 |                 |                 |                 |
|-----------------|-----------------|-----------------|-----------------|
| (a) $(a - b)^2$ | (b) $(1 + x)^2$ | (c) $(2 - x)^3$ | (d) $(6 + h)^3$ |
|-----------------|-----------------|-----------------|-----------------|

7. Simplify  $\frac{(6 + h)^3 - 216}{h}$  if  $h \neq 0$ .

8. If a point on the line

$$y = -\frac{1}{3}(x - 6) + 8$$

has an  $x$ -value of 15, what is its  $y$ -value?

9. If a point on the line

$$y = -\frac{1}{3}(x - 6) + 8$$

has an  $x$ -value of 6, what is its  $y$ -value?

10. Graph each of the following:

(a)  $y = 3(x - 1) + 2$

(c)  $y - 2 = 3(x - 1)$

(e)  $3x - y = 1$

(b)  $y = 3x - 1$

(d)  $y + 1 = 3x$

(f)  $x = (y + 1)/3$

11. Give an example of a point that is on the line

$$y - 17 = 38(x - 12).$$

12. Describe the shape of  $y = 7$  in words. Describe  $x = -2$  in words.

13. Give an equation for the line through the point  $(-6, 5)$  with slope 2.

14. Give an equation for each of the following:

(a) the line through  $(1, 3)$  with slope 5.

(b) the line through  $(0, -9)$  with slope  $\frac{2}{5}$ .

(c) the line through  $(-4.2, 6.1)$  with slope 8.88.

(d) the line through  $(5, 1)$  with slope  $-3$ .

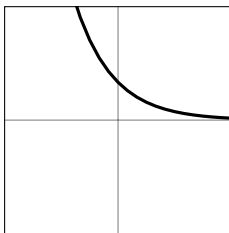
15. A line passes through both  $(4, 4)$  and  $(8, 2)$ . What is its slope?

16. Give an equation for the line through  $(1, 7)$  and  $(10, -6)$ .

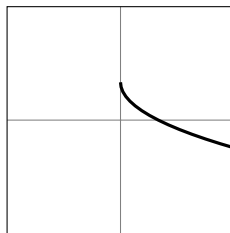
17. For  $f(x) = \begin{cases} x + 1 & \text{if } x < 4 \\ x^2 & \text{if } x \geq 4 \end{cases}$ , what is the value of  $f(4)$ ?

18. Draw a graph of the piecewise function  $\begin{cases} x & \text{if } x < 1 \\ 2x & \text{if } 1 \leq x \leq 2 \\ 6 - x & \text{if } x > 2 \end{cases}$ .

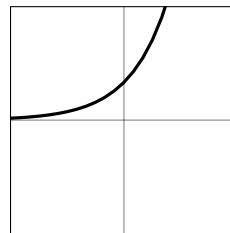
19. Match the function with its graph: (a)  $e^x$  (b)  $\sqrt{x}$  (c)  $1 - \sqrt{x}$  (d)  $e^{-x}$



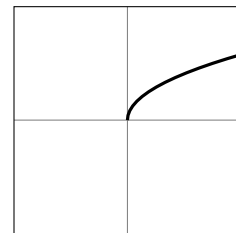
(I)



(II)



(III)



(IV)